**MINISTRY OF EDUCATION AND TRAINING**

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**VU THI BINH**

# FOSTERING MATHEMATICAL REPRESENTATION AND MATHEMATICAL COMMUNICATION COMPETENCIES FOR STUDENTS IN TEACHING GRADE 6 AND 7 MATHEMATICS

**Major: Theory and Method of teaching Mathematics**

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**THE RELATED SCIENTIFIC WORKS PUBLISHED BY THE AUTHOR**

[1]. Vu Thi Binh (2013). *Mathematical clause, Mathematical theorem and language form to display them in Geometry for Grade 6.* Journal of Educational Science, Special issue, October 2013.

[2]. Vu Thi Binh (2013). *Exploitation of language factors by practicing and dividing two exponents with the same base - Grade 6 mathematics*. Journal of Educational Science, Installment 2, November 2013

[3]. Vu Thi Binh (2014). *Mathematical terms and mathematical symbols in teaching concepts of mathematics; review and further natural numbers for Grade 6.* Journal of Educational Science, Special issue, January 2014.

[4]. Vu Thi Binh (2014). *Some issues in mathematical communication and representation in teaching mathematics in secondary schools.* Journal of Educational Science, Special issue, October 2014.

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# PREAMBLE

## 1. REASON OF STUDY

1.1. Mathematics is an important subject at secondary schools and mathematical language has significant meaning in mathematics education. The mathematical language has become a characteristic of modern mathematical thinking, playing the important role in the development of mathematics awareness. Thereby, paying attention to the mathematical language in teaching mathematics is an obvious issue.

Studies on the mathematical language in mathematics education at secondary schools in Vietnam have brought about many important results, expressing in documents which are used for training and improving capacities of teachers. The doctoral thesis by Tran Ngoc Bich, Thai Huy Vinh, and Hoa Anh Tuong have continually affirmed that the mathematical language is the important factor which contribute to increase the students' results of learning mathematics. Obviously, the study, exploitation and use of mathematical language regarding the establishment and development of the students' mathematical competency have been more and more meaningful.

1.2. Tendency of competency development in school education worldwide and requirements on innovation of school education in Vietnam currently. The school education program in many countries worldwide has determined clearly fundamental aspects, competencies and requirements of qualification and attitude. The strategy of education development in Vietnam in 2011 – 2020 determines that the competency of student is the important orientation to develop curriculum and textbooks upon 2015. Study made by Niss Mogens on mathematical competency and Programme for International Student Assessment (PISA, 2009) are applied to determine 8 competencies of knowing the mathematics to 15-years-old students. In which, the mathematical communication and mathematical representation are two important competencies.

1.3. The opinion of teaching which establishes the mathematical competency of students through activities and learning activity has been affirmed by many mathematics educators. The innovation of student-centered teaching method has realized in schools. However, by Nguyen Huu Chau, it can be said that "there are not many proofs which show the considerable change of teaching methods". Actually, the students have still faced with many difficulties in communication and self representation of mathematical contents. Many teachers have not applied measures which attract the students to participate in learning activities in general, the mathematical representation and mathematical communication in particular. The students lack the sense of initiative, confidence, environment and motivity to involve in learning activities. The establishment and organization of cases for the students to apply the mathematical representation and mathematical communication are not only the premise which stimulates such activities but also contribute to clarify the oriented innovation of teaching method which develops the learners' competency, increases the learners' responsibility regarding the establishment of their mathematics understanding and taking the initiative to form their firm knowledge, establish and develop the capacity of connecting the mathematics with reality. In the context of innovation of school education, the research and establishment of measures which improve the students' competencies of mathematical representation and mathematical communication in teaching mathematics have been more and more essential in order to establish and develop the competency and qualifications of learners.

From above reasons, we have made the research titled: ***Fostering mathematical representation and mathematical communication competencies for students in teaching grade 6 and 7 mathematics.***

## 2. Overview of researched issue

### *2.1. International*

*a. Opinion on language in mathematics education.* From the mid 20th century, the Soviet researchers of mathematics education have been much interested in the language in teaching mathematics at schools. Explaining the students' formalism in learning mathematics, Khinxin thought that "*in the mind of students, there is any disruption against the mutual and reasonable relationship between the internal content of mathematical event and external representation of such event (in oral, signal or visual images)*". A.Xtolyar has also noted that both semantics and syntax of the mathematical language are very important and pedagogic problem on reasonable balance between such two aspects has profound methodological meaning.

Recently, Congress of European Research in Mathematics Education (CERME1,1999), (CERME4, 2005) has focused on teaching, developing the mathematical language in the aspects of lexicology, syntax and semantics. Clare Lee (2006), Chad Larson (2007), Shelly Frei (2008) have shown the role of mathematical language and suggested the teaching methods for the students to grasp thoroughly the mathematical language. With the tendency of teaching to develop competencies, the researches have paid more and more attention to the use of mathematical language in the mathematical representation and mathematical communication of students.

b. *Results of research on mathematical representation and the development in mathematics education.* J. Bruner, a US psychology has shown that there are three representation forms of a subject: (a) actions, (b) images (models, diagrams), and (c) language symbols, mathematical clauses and theorems. Thence, there are three respective learning activities of learners (1) *Analysis of certain thing (by hand); (2) modeling* and *(3) symbolization (symbols*). On such basis, Clark & Paivio determined two systems of *verbal* and *image* representation. Marzano, Pickering and Pollock took into account of *verbal* and *non-verbal* representation. Lesh, Landau and Hamilton showed five types of representation: *Experiences in real life; Models of operation; Images or diagrams; Words; Writing symbol*. Tadao gave 5 types of representation which relate with each other in teaching mathematics: *Real representation; Representation by operative model; Illustrative representation by images (visual representation); Representation by language; Representation by symbols.* Previously, in curricula of mathematics at schools, the mathematical representation was often considered as a part of mathematical communication. However, the tendency which considers the mathematical representation as an independent competency from the mathematical communication has been more and more interested in. Some mentioned works are: "*Role of representation in mathematics at schools",* "*Representation and Visual Mathematics",*.... In 2000, NCTM made the representation and communication become two of five standards under the process of mathematics curricula at schools. Thence, the mathematical representation has been the compulsory standard in teaching and assessing the mathematics at schools in US and some countries in the world.

*c. Development of mathematical representation and mathematical communication competency for students in teaching mathematics.* Researchers of mathematics education have been more and more interested in the establishment and development of mathematical language for students through learning activities, particularly activities of mathematical communication by mathematical language. In "Key strategy of developing mathematical vocabulary at secondary schools", Rheta N. Rubenstein thinks that, the communication must be an important content of mathematics education and the learning of vocabulary must be mentioned to as an effective tool of mathematical communication. Research of Glenda

Anthony and Margaret Walshaw has shown that the mathematical communication, mathematical language and tools of mathematical representation are three of ten basic principles of innovation of teaching mathematics. According to Emori Hideyo: "All experiences of mathematics are done through communication. The mathematical communication is necessary to develop the mathematical thought because the development of thought is explained by language of subject bad ways of communication". The mathematical communication and mathematical representation have become two core competencies which are necessary to develop for students in standard curricula of mathematics in US.

The conference on innovative method of teaching mathematics held by APEC (Thailand, 2008) and the 36th Conference of International Group for the Psychology of Mathematics Education (PME 36, Taiwan, 2012) have shown the impacts and benefits of mathematical communication in teaching and learning, and concluded that the mathematical communication is the important component of mathematical competency framework.

Nowadays, the opinion which considers the mathematical communication and mathematical representation as core issues which are required in the curricula of mathematics at schools has been recognized in many countries in the world such as New Zealand, Rumania, Australia, US, Germany, Denmark… However, by now, we have still not approached any specific and systematic researches on the determination of mathematical representation and mathematical communication activities associated to the content of teaching mathematics and measures of improving such competencies for students in teaching.

**2.2. In Vietnam**

*a. Results of research on mathematical language.* Vietnamese mathematics educators such as Pham Van Hoan (1981), Hoang Chung (1995), Pham Gia Duc, Vu Quoc Chung, Do Trung Hieu, Do Dinh Hoan, Ha Si Ho (1992), Nguyen Ba Kim, Vu Duong Thuy (1992),.... have paid considerable attention to the mathematical language in teaching mathematics at schools in documents which are used for training and improving mathematics teachers. Authors such as Nguyen Ang, Do Tien Dat, Dao Thai Lai, Pham Thanh Tam and Nguyen Tuan (2004) have analyzed the mathematical language in teaching mathematics at primary schools. Ton Nu Mi Nhat (2013) and Le Van Hong (2013, 2014) suggested the way of approaching language in teaching mathematics. In addition, many researches have affirmed the role of mathematical language in teaching mathematics at schools, by Bui Huy Ngoc; Tran Anh Tuan; Chu Cam Tho; Phan Anh Tai; Nguyen Van Thuan; Tran Ngoc Bich; Thai Huy Vinh,...

*b. Mathematical competency in teaching mathematics at schools.* Works of V.A. Krutexki translated by Pham Van Hoan, Hoang Chung into Vietnamese have initially marked the researches on mathematical competency in Vietnam. Therefrom, Tran Luan (1996); Tran Dinh Chau (1996) have done researches on specific mathematical competencies. Recently, in the research which is aimed at realizing the orientation of developing the competency of learners in teaching mathematics, there are some additional important results: Do Tien Dat (2013); Tran Kieu (2014); Do Duc Thai (2014); Nguyen Ba Kim (2015); Pham Duc Quang (2016) ; Chu Cam Tho (2014)...

*c. Mathematical representation and mathematical communication competencies in teaching mathematics.* The new program of school education after 2015 determines that the communication (in Vietnamese) is one of general core competencies. The mathematical communication is determined as one of six mathematical competencies at schools, in which the representation is one of factor of mathematical communication. Taking interest in visual representation, Tran Vui confirmed: The visual representation is not only the illustrative

means but also the tool which supports the students' process of thinking… Visual representation must be recognized as the main component of reasoning and requires additional researches in teaching mathematics at schools. The research, establishment and development of mathematical representation and mathematical communication competency for students have attracted the interest by many authors at different level. Some results of researches are: Author Phan Anh affirmed that the competency of using the natural language and mathematical language are the premise of mathematical competency becoming actual case of high school students. Tran Ngoc Bich, Thai Huy Vinh mentioned the mathematical communication skills as a measure of increasing the efficiency of using mathematical language for primary school students. Hoa Anh Tuong paid attention to *"Using and researching lessons to foster the mathematical communication competency for secondary school students".* In which, the mathematical representation is determined as one of basic modes of mathematical communication and it is suggested to apply ways of teaching mathematical problems by open ending to foster the process of mathematical communication. Nguyen Thi Tan An uses the mathematics to develop the competency of quantitative understanding, thereby developing the mathematical *representation* and mathematical *communication* competency (which are two components of competency of quantitative understanding).

Therefore, by now, in Vietnam, researches on measures of improving the mathematical representation and mathematical communication competency for students through specific mathematical representation and mathematical communication associated to the content in teaching mathematics. The establishment and development of mathematical communication and mathematical representation for students are mainly through solving mathematical problems (mathematical cases, open-ending mathematical problems). Therefore, there are many issues which require additional interest and researches regarding the improvement of mathematical representation and mathematical communication for secondary school students in general and students of grade 6 and 7 in particular in the direction of determining and organizing for students to realize effectively the mathematical representation and mathematical communication activities in teaching mathematics.

**3. Purpose of research:** Recommend pedagogic measures to foster mathematical representation and mathematical communication competencies for students in teaching grade 6 and 7 mathematics.

**4. Object, entity, scope of research**

***4.1. Object:*** Process of teaching mathematics at secondary schools

***4.2. Entity:*** Fostering mathematical representation and mathematical communication competencies for students in teaching grade 6 and 7 mathematics.

***4.3. Scope:*** The thesis focuses on the exploitation and use of mathematical language, including signs, terms and mathematical symbols (drawings, charts, diagrams,…) in order to foster mathematical representation and mathematical communication competencies for students in teaching grade 6 and 7 mathematics.

**5. Scientific assumption:** In teaching grade 6 and 7 mathematics, if the establishment and implementation of measures of fostering the mathematical representation and mathematical communication are based on the determination and helping students practice any specific mathematical representation and mathematical communication activities, the mathematical representation and mathematical communication competencies and students' results of mathematics learning shall be improved.

**6. Duties of research:** Theoretical research of mathematical language, mathematical representation and mathematical communication in teaching mathematics at secondary

schools; Research of mathematical language included in curricula and textbook of grade 6 and 7; Actual conditions of teaching and fostering mathematical representation and mathematical communication competencies for students in teaching grade 6 and 7 mathematics; Pedagogic practice and initial assessment of feasibility and effectiveness of recommended measures.

**7. Methods of research**

***7.1.Theoretical method:*** Analyze, summarize, collect information, research documents, … to establish theoretical foundation of the topic.

***7.2. Practical method:***

*Method of observation, survey and interview:* Survey the actual situation of teaching and using the mathematical language, mathematical representation and mathematical communication for students in teaching mathematics at secondary schools. Observe students in learning mathematics at secondary schools. Interview and survey the organization of activities in order to establish and develop the mathematical representation and mathematical communication competencies for students of grade 6 and 7.

*Method of experts:* Apply for ideas of experts and researchers on issues to the extent of the researched topic.

*Method of product research:* Examine notebooks, tests and study cards of students to find out their competencies of mathematical representation and mathematical communication in learning mathematics at secondary schools.

*Method of pedagogic practice*: Pedagogic practice is aimed at checking the feasibility and effectiveness of recommended pedagogic measures.

*Method of mathematical statistics:* Process surveyed and actual data.

**8. Theoretical and practical significance**

*Theoretically,* clarify opinions, components, specific expression and levels of mathematical representation and mathematical communication competencies of students in learning mathematics at secondary schools; Determine scientific basis of measures of fostering mathematical representation and mathematical communication competencies for students in teaching grade 6 and 7 mathematics.

*Practically*, recommend some measures of fostering mathematical representation and mathematical communication competencies for students in teaching grade 6 and 7 mathematics.

**9. Presented contents**

- Opinion of mathematical representation and mathematical communication and specific mathematical representation and mathematical communication activities of students in learning mathematics at secondary schools;
- Opinion of mathematical representation and mathematical communication competencies, components, specific expression and levels of mathematical representation and mathematical communication competencies of students at secondary schools;
- Pedagogic measures of fostering mathematical representation and mathematical communication competencies for students in teaching grade 6 and 7 mathematics.
- Results of pedagogic practice.

**10. Layout of the Thesis:** In addition to the Preamble and Conclusion, the Thesis includes 3 chapters.:

**Chapter 1:** *Basic theory and practice*; **Chapter 2:** *Measures for fostering mathematical representation and mathematical communication competencies for students in teaching grade 6 and 7 mathematics.* **Chapter 3:** *Pedagogic experimentation*

# Chapter 1
# BASIC THEORY AND PRACTICE

## 1.1. Competency, mathematical competency for secondary school student and fostering mathematical competency.

### 1.1.1. Concept of competency

Competency is a concept of psychological categories. This includes the concept of competency provided by Nguyen Cong Khanh, Québec Ministry of Education, ... In addition, Xavier Roegiers (1996) confirms, "*competency is integration of skills impacting naturally on contents in a given situation in order to solve the problems posed by this situation".* The concept of competency provided by Xavier Roegiers is close to education and in accordance with research of the thesis. It can see that conceptions of competency have uniformity as follows:

*For features*: Competency is formed and exposed in its activities; *Regarding relations with the knowledge, skills*: Knowledge, skills are a necessary condition to forming competency which contributes to the knowledge and skill comprehending process.

### 1.1.2. Mathematical competency for secondary school student

Concept of mathematical competency for secondary school student from research of V.A Krutexki, up to now has had significant changes and development.

Niss Mogens has provided the concept of mathematical competency selected by PISA. Accordingly, PISA 2015 thinks that Mathematical literacy *is the ability of individuals who know to formulate, employ and explain mathematics in many contexts. It covers mathematical reasoning and use of concepts, methods, events, and mathematical tools to describe, explain and predict phenomena. It helps people to recognize the role of mathematics in the world and make judgments and decisions of citizens for contribution of their idea, participation and reflection*". This is also the concept of mathematical competence used in the research of the thesis. Many programs in mathematics at the secondary school in the world such as USA, Australia, Canada, New Zealand, Singapore, Germany, Denmark, ... have been specified according to content and process in order to determine the elements of mathematical competency, to establish mathematical competency for student in teaching and assessment.

### 1.1.2. Fostering mathematical competency for student

According to the Vietnamese dictionary edited by Hoang Phe, *Fostering 1*. Make increase of health with nutrients, 2. Make increase of qualifications, competency or quality. According to dictionary edited by Nguyen Nhu Y, *Fostering 1*. Make more healthy, more strength, 2. Make better, more proficiency.

It may provide general that, *the fostering mathematical competency for student is the process for organization for students to apply mathematical knowledge, skills to perform learning activities compatible with the elements and specific expression of each competency. Thereby the competency of student shall be developed higher.*

## 1.2. Competency in using mathematical language

### 1.2.1. Overview of mathematical language

#### *1.2.1.1. Concept of mathematical language*

Mathematics educators in Vietnam have paid deeper and more sufficient attention to mathematical language such as Pham Van Hoan, Ha Si Ho, Hoang Chung, Nguyen Ba Kim; From the researches provided by Tran Anh Tuan, Le Van Hong, Tran Ngoc Bich, Thai Huy Vinh, it can provide the following general: *The mathematical language in teaching mathematics for secondary school student is the language of mathematical science, including mathematical terms (word, phrase), mathematical symbols (e.g. drawings,*

*diagrams, graphs…) and their association rules used for describing objects and mathematical relationships during speaking, writing or thinking.* Of which:

*Symbols* include numbers, letters, alphabetic characters, arithmetic signs, relation signs, and vocabulary signs and brackets used in mathematics.

*Mathematical terms* include words and phrases which are names of concepts, objects and relations in the mathematical fields (e.g. prime numbers, composite, line, vertical opposition, power,...); words, phrases of mathematical language, but in mathematics, they have characteristic meaning (e.g. edge, centre, denominator, numerator, ...). As well as scientific term in general, mathematical terms do not bear expressive rhetorical nuance, they have determined, systematic, simplicity and international significance

*Mathematical symbols* include photographs, drawings, diagrams, graphs or models to represent the mathematical relationship and the specific mathematical objects.

***1.2.1.2. Characteristics of mathematical language***

Mathematical language is creation result of human to express the mathematical facts, is the remedy of mathematical language according to tendency: *Remedy cumbersome of the mathematical language; Expand expression capacity; Eliminate ambiguity of the mathematical language* (Phan Anh). In Pham Van Hoan's opinion, the mathematical language has important characteristics: *Succinctness; Accuracy expression capacity of mathematical ideas; Capacity of overview and expression of general rules*. Moreover, with notion of thesis about the mathematical language, including drawings, diagrams, charts, graphs,... which shows that the "intuition" of the mathematical language is advantage, brings tremendous advantages for thinking and in exchanging, communicating mathematical ideas.

***1.2.1.3. Function of mathematical language***

*a. Communication; b. Thinking*

**1.2.2. Mathematical language activities in teaching mathematics**

***1.2.2.1. Concept of mathematical language activities***

F.Sausuare defines the concept of *language* in the distinction with *parole* and *language activities*. Pham Minh Hac noted: "*Language is a process that each individual uses a language for exchange*." Nguyen Quang Uan described "*parole activity* when performing communication purposes or when thinking is actually the process of forming and expressing idea through language". Nguyen Ba Kim paid attention to the *language activity* as one of five main study activities of students. In our opinion: *The mathematical language activities in mathematics class are the teaching activity in which the teacher and students use the mathematical language and natural language to* ***exchange, communicate, think, present, express and receive*** *ideas, views, contents of mathematics.*

The thesis focus on the mathematical language activity in teaching mathematics in the secondary school, in the sense:

- Students are subjects to perform activities, associated with mathematical contents and language (Vietnamese and mathematical language) which are means and result of such activities.

- The mathematical language activity focus on *exchange, communication, thinking, presentation, expression and receipt* of ideas, views, mathematical content, development of thinking and communication function of the mathematical language (and including natural language) in teaching mathematics.

The following will clarify several mathematical language activities in teaching mathematics.

***1.2.2.2. Mathematical language activities in teaching mathematics in secondary school***

*a. Receive mathematical language on vocabulary, syntax and semantics in correct,*

*logic, systematic manner*

*b. Transfer idea into word (mathematical language) for thinking and communication*

*c. Transfer language from various types of the mathematical language, "translate" natural language into mathematical language and vice versa.*

**1.2.3. Competency in using mathematical language**

***1.2.3.1. Competency in using language:*** According to Vietnamese dictionary, "Use" means "Take something as a mean to serve the needs of a certain purpose". " "Use language": Using language as a mean to serve the implementation of language activities. Thus, it can show that *competency in using language is ability to master the knowledge, skills of language to effectively implement language activities in specific contexts.*

***1.2.3.2. Competency in using mathematical language***

On the basis of analysis of language, mathematical language, and related concepts, in our opinion, *the competency in using mathematical language of the student is the ability to master and apply effectively the mathematical language to successfully implement language activities in the mathematical research and study process as well as in social life in general.*

The competency in using mathematical language includes: *(1) Ability in receiving and understanding knowledge, skills of mathematical language; (2) Ability in establishing, applying and practice effectively the mathematical language in communication and thinking; (3) Ability in selecting, changing language in study and practice*.

**1.3. Mathematical representation competency**

**1.3.1. Mathematical representation**

***1.3.1.1. Concept of mathematical representation***

According to Vietnamese words and expressions dictionary, *representation*: *"writing in pictures or symbols";* some pages of online dictionary also describes *representation*: "*Expressing by formula or drawings*" (Tra Tu Dictionary)*; "Expressing by symbols or drawings"* (Lac Viet dictionary), ... We think that, *the mathematical representation is the use, arrangement of terms, symbols, images (diagrams, charts, drawings, graphs, signs on paper, geometric sketch, ...) or specific objects containing mathematical content* ***to describe, symbolize or represent*** *an object, a relationship or mathematical procedure.*

The above concept shows that: The mathematical language includes representations on actual objects (objects, relationships in natural-social life), visual representations (using diagrams, charts, tables, specific images, ...) and language representation (terms, formulas, mathematical symbols ...). Visual representation plays an intermediary role in connecting actual representation with representation symbols.

***1.3.1.2. Classification of mathematical representation***

*a. Representation by convention and not convention:*

*b. Internal representation and external representation:*

*c. Classification under use forms of mathematical representation*

**1.3.2. Mathematical representation activities in learning mathematics at secondary school**

At secondary school, the role of mathematical representation is focused on exploiting in deep and diversified manner to discover the rules, relationships, mathematical relations. The students do not only simulate, illustrate, but also use the flat, shapes, diagrams, graphs, charts, .. to help generalization of general rule, abstract relations, application of the mathematical representations in research, explanation of phenomena of the life. On the basis of research results provided Tadao regarding the mathematical representation types and consideration of the mathematical representation activities on mathematical representation

aspect, the thesis identified the mathematical representation activities in learning mathematics of the secondary school students, including:

*1.3.2.1. Activity of knowing and understanding the mathematical content of the mathematical representations in accurate, logic, systematic manner (decode activity)*

*1.3.2.2. Activity of associating, changing or creating the mathematical representations suitable to situations, specific contexts. (code generating activity)*

*1.3.2.3. Activity of selecting, changing the mathematical representations in process of awareness, practice, memorization and mathematical communication. (activity of code selection and conversion)*

**1.3.3. Mathematical representation competency**

***1.3.3.1. Concept of mathematical representation competency***

Application of above research result of mathematical representation, consideration of mathematical representation competency is a format of the competency in using the mathematical language, having interrelation with the mathematical communication competency, in our opinion, *the mathematical representation competency is the ability in* ***understanding, using, selecting, creating and converting*** *the mathematical representations to think, memorize, describe, explain, make argument, connect and exchange ideas in solving mathematical issues.*

***1.3.3.2. Specific expressions of mathematical representation competency.*** We determine 3 components and specific representations of the mathematical representation competency, including:

| ***Components*** | ***Specific representations*** |
|---|---|
| *1. Understand and use effectively the mathematical representations to think, memorize or present the mathematical contents* | 1.1. Discriminate, properly understand the contents of the objects and mathematical relations in the mathematical reresentations.<br>1.2. Use the mathematical representation system to think, memorize or present the mathematical content. |
| *2. Associate, change or create suitable mathematical representations to search idea, solution or solve mathematical issues* | 2.1. Be able to associate, change representations to connect, make argument, prove; search solution, mathematical idea.<br>2.2. Create suitable mathematical representations to show objects, relations or method for solving mathematical issues in various situations. |
| *3. Select, convert favorable mathematical representations for awareness, practice, memorization and mathematical communication.* | 3.1. Select reasonable mathematical representation method in diversified study circumstances.<br>3.2. Convert favourable mathematical representation forms for awareness, practice, memorization and mathematical communciation.<br>3.3. Translate natural language into mathematical representations to modelize, suitable to specific context, create effectiveness in thinking and communication. |

**1.3.4. Levels of mathematical representation competency:** Including 5 levels from low to high.

**1.3.5. Mathematical representation competency and result of learning mathematics of student**

## 1.4. Mathematical communication competency

### 1.4.1. Mathematical communciation

#### *1.4.1.1. Concept of communication and communication in teaching:*

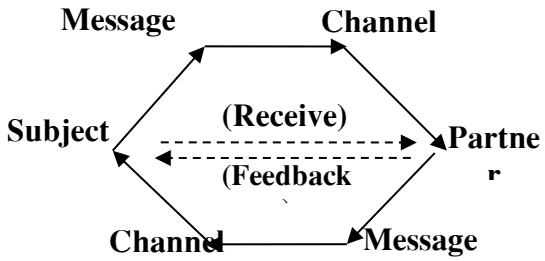



**Figure 1.14:** *Communication phase in teaching*

In teaching, interaction, contact of communication may analize communication process into *communication phases.* On the basis of communication diagram introduced by Nguyen Huu Chau, the thesis specificalizes elements of one *communication phase* in the teaching process. Each *communication phase* has *subject* encoding content into *message,* transmiting through communication *channel* (speech, handwriting, lời nói, chữ viết, slides, ...) orienting *partner* (communication co-subject). Meanwhile, *partner receives* message, *opens code* and *provide feedback* (with *messages* and *communication channel* of the partner) to communication subject (Figure 1:14).

#### *1.4.1.2. Mathematical communication*

In view of social construction in teaching, Paul Ernest said that "the objective knowledge is constructed by individual through their interration relationships with teachers and classmates to constitute personal subject knowledge". Obviously, construction, occupation of mathematics by student are always closely associated with mathematical communication activities in learining mathematics. On this basis, the concept of *mathematical communication* mentioned in the thesis *is the communication that takes place between teacher-student, student-student in the process of teaching marthematics. This process uses the mathematical language as important and main mean to receive and transmit mathematical ideas, mathematical knowledge, provide arguments, prove, solve issues in order to achieve goal of learning mathematics*.

### 1.4.2. Mathematical communication activities in teaching mathematics

In opinion of L.X Vygotsky, the formation of cultural psychological functions occurs in the interaction between individuals. From main theoretical points of historical-cultural theory of L.X Vygotsky, which has created modern teaching model: Teaching interacting development with an important characteristics shows the meaning and role of communication activity in teaching. Accordingly, the mathematical communication activity is considered a major activity so that the students "learn how to put symbol tools into inside and learn how to use them, turn them from external social means into internal mean, psychology of the individual". The mathematical communication activity includes:

*1.4.2.1. The communication activity receives mathematical knowledge, skills through listening comprehension, reading comprehension and notes by the mathematical language.*

*1.4.2.2. Communication activity creates the oral or written mathematical language in presentation of mathematical solutions, ideas.*

*1.4.2.3. Communication activity interacts in case of exchange, discussion, convincement, explanation and assessment of mathematical ideas, solutions by the mathematical language in exchange with friends, teachers*.

### 1.4.3. Mathematical communication competency

#### *1.4.3.1. Concept of mathematical communication competency*

Consistent with the concept of mathematical communication and 3 types of mathematical communication activities mentioned above, we think that *the mathematical communication competency is the ability in understanding mathematical issues through written, oral, graphic communication; ability in effectively use the mathematical language in close relationship with the natural language to exchange, present, explain, make argument, prove mathematics in accurate, logic manner, clarify mathematical ideas in specific context.*

#### *1.4.3.2. Specific expression of mathematical communication*

For more convenience for fostering the mathematical communication competency, the thesis determines 3 components and specific expression of the mathematical communication as follows:

| Components | Specific expressions |
|---|---|
| *1. Receive, comprehend mathematical knowledge, skills by comprehensive listening, reading and writing.* | 1.1. Comprehensive listening, reading and summarize in writing basic elements, focus of mathematical contents, requirements.<br>1.2. Capable of making questions to clarify learning demands, tasks.<br>1.3. Correctly understanding question and information relating to learning tasks in specific circumstances. |
| *2. Create verbal and writing mathematical products to present mathematical ideas, measures during the learning process.* | 2.1. Fully, exactly, logically present mathematical contents, ideas<br>2.2. Coherently explaining thoughts about mathematical approaches, variation and the basis. |
| *3. Effectivel use mathematical and natural language in exchanging, discussing, finding measures; persuading, explaining and evaluating mathematical contents, ideas in interaction with classmates, teachers.* | 3.1. Appropriately combine, transfer, use mathematical presentation and communication in developing, finding approaches for mathematical issues, tasks and situations.<br>3.2. Analyze, compare and choose appropriate mathematical ideas, approaches.<br>3.3. Convincingly present, closely agrue, show self-confidence, respect (listeners, speakers) in describing, explaining mathematical contents, ideas. |

**1.4.4. Levels of mathematical communication competency**: Including 5 levels from low to high.

**1.4.5. Students' mathematical communication competency and learning results**

### 1.5. Mathematical communication and presentation in relation with natural language using competency.

#### 1.5.1. Relation between mathematical language competency and mathematical communication and presentation competencies

Niss Mogens identified mathematical communicatin and presentation competencies *are under competencies group: the ability to deal with mathematical language and tools* (figure 1.15). The relation between natural language competency and mathematical communication & presentation are considered as a relation between the whole and parts, between the general and the particular. Without researching the general, the particular could not be understood and vice versa.

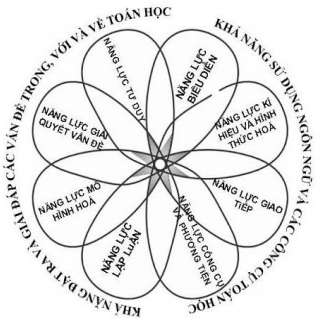



Figure 1.15

**1.5.2.** The relationship **between mathematical presentation and communication competencies**

**1.6. Current situation in teaching and fostering mathematical presentation and communication competencies in teaching Mathematics at Junior-High School level.**

**1.6.1. Junior-High School students' thought and language development**

**1.6.2. Characters of mathematical language in mathematical textbook for students at grade 6 and grade 7**

*1.6.2.1. Terms and signals are in proportion to contents of mathematical textbook for students at grade 6 and grade 7.*

*1.6.2.2. Mathematical symbols in mathematical textbooks for students at grade 6 and grade 7.*

**1.6.3. Survey of real situation in fostering mathematical presentation and communication competencies in teaching Mathematics at Junior-High School level.**

***1.6.3.1. Purpose of survey:*** Understanding about real situation in teaching and fostering mathematical presentation and communication competencies for students of junior-high schools.

***1.6.3.2. Subjects of survey:*** 438 teachers of 48 Junior-High Schools in Lao Cai, Yen Bai, Hanoi, Thai Binh; in-service students of mathematical inter-college program at Hanoi Pedagogical University 2, Thai Nguyen University of Education; 1900 students of Junior-High Schools in Lao Cai, Yen Bai, Hanoi and Thai Binh were surveyed.

Of which, about 70% of survey notes focusing on teachers of grade 6 and grade 7 (300 survey notes) and students of grade 6 and grade 7 (1300 survey notes).

***1.6.3.3. Contents of survey:*** Understanding about the appropriateness of mathematical language in junior-high school mathematical textbook; the understanding, application of students' mathematical language; the teaching for fostering mathematical presentation and communication for students in teaching mathematics at junior-high school level, especially at grade 6 and grade 7.

***1.6.3.4. Measures for survey:*** Investigation, conversation, interviewing, observation, product research, statistical data processing were used for the survey.

***1.6.3.5. Outcomes of survey:*** Survey outcome has showed that most of teachers, managers have been aware of the role, importance of fostering students with mathematical presentation and communication competencies during the teaching process. Results of students' mathematical presentation and communication competencies were poor. Mathematical presentation and communication skills training have been unspecific, formal. Teachers have faced many difficulties in choosing contents, identifying appropriate measures and organizing to implement teaching and developing students' mathematical presentation and communication competencies. These should be researched to propose appropriate, feasible measures to effectively foster the above competencies.

***1.6.3.6. Reasons***

(1) Yet teachers have not really considered mathematical presentation and communication are learning activities and not fully identified activities of mathematical presentation and communication as well as typical symbols of these competencies to organize compatible teaching activities.

(2) Yet teachers have not provide measures for implementing mathematical presentation and communication fostering measures to students attaching to contents of mathematical programs.

(3) As teaching conditions must satisfy strict requirements for the goals, contents, knowledge, skill, time, evaluation, etc... Teachers have faced many difficulties in identifying and exploiting opportunities to organize to provide mathematical presentation and communication competencies to students during the teaching process.

**Conclusion of Chapter I**

The following issues have been researched and clarified by Chapter I:

*Firstly,* analyzing, understanding about research outcomes relating to competency, mathematical literacy competency. Consent of competent approach which is a set of skills and student competency fostering should be performed by effective organization of learning activities compatible to that competency.

*Secondly,* basing on mathematical language, mathematical language applying activities, we have identified 3 mathematical fostering activities and 3 mathematical communication specific for teaching mathematics; introduced opinions on mathematical presentation competency and mathematical teaching competency in teaching mathematics at junior high school level; described typical components, symbols and identified 5 levels for those competencies; As a result, both teachers and students can identify, organize to implement, observe, evaluate mathematical fostering teaching activities during the teaching process.

*Thirdly,* focusing on analyzing contents of program, grade-6 and grade-7 mathematical textbooks in the aspect of mathematical language. Finding about real condition of teaching, fostering mathematical presentation and communication competencies. Analyzing principles reasons leading to that real condition.

The above research outcomes have showed that research for proposing measures for fostering and developing mathematical representation and communication competencies has a critical, scientific and practical meaning in the context of innovating teaching by competencies approaches contributing to upgrading effectiveness of mathematical teaching.

# Chapter 2
# MEASURES FOR FOSTERING MATHEMATICAL REPRESENTATION AND MATHEMATICAL COMMUNICATION COMPETENCIES FOR STUDENTS IN TEACHING MATHEMATICS OF GRADE 6 AND 7

## 2.1. Orientation of making measures for fostering mathematical representation and mathematical communication competencies for students in teaching mathematics of grade 6 and 7.

### 2.1.1. Ensuring the compliance with the object, content and knowledge standard, skill of the mathematics program.

### 2.1.2. Focusing on the characteristics, role and position of mathematical language in the close relationship with mathematical language during organizing activities of mathematical representation and mathematical communication.

### 2.1.3. Grasp thoroughly the opinion of activity in establishing and developing the mathematical representation and mathematical communication competencies

## 2.2. Measure group 1: Fostering mathematical representation competency

**2.2.1. Measure 1.1** *Organizing activities for the students to know, understand and properly use the mathematical representation forms on subjects, relations and steps of mathematical changes.*

*2.2.1.1. Object of the measure.*
*2.2.1.2. Scientific ground of the measure*
*2.2.1.3. How to take measure*

a. Teacher should fostering by itself during teaching process.

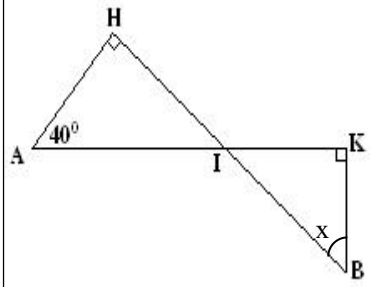
b. For mathematical representation by convention, there are 3 steps: *Introduction, description*; *identification, demonstration*; *application.*

c. For mathematical representation not by convention, teacher may allow students to decode by themselves or teacher keep them informed of the process and help students practice activity components compatible with mathematical representation process.

*2.2.1.4. Attention during taking measure*
*2.2.1.5. For example*

**Example 2.7.** See figure 2.5. Find x? (Maths for Grade 7, volume 1, page.109)

***Comment:*** Teacher help students practice "diagram for finding and guess"

| ***Teacher suggests – students answer*** | ***Teacher writes on the blackboard – students look at and imitate*** | ***Figure*** |
|---|---|---|
| From the figure (supposition), we see that:<br>- Δ IKB perpendicular at K, so in order to calculate angle x, which we need to know? ($\widehat{BIK}$ =?)<br>- In order to calculate $\widehat{BIK}$ , which angle do we need to calculate? why? ( $\widehat{AIH}$ =?, because $\widehat{AIH}$ vertically opposite to $\widehat{BIK}$ )<br>- In ΔAIH, we have known that:<br>$\widehat{IAH} = 40^0$ (figure); $\widehat{AHI} = 1v$ (figure)<br>- So: $\widehat{AIH}$ =? Why? | Diagram for finding and guess<br>(*from bottom to top*)<br>$\widehat{IAH} = 40^0$ (figure)<br>$\widehat{AHI} = 1v$ (figure)<br>⇑<br>$\widehat{AIH}$ =?<br>⇑<br>$\widehat{BIK}$ =?<br>⇑<br>**x=?** |  <br>Figure 2.5 |

Next, students will present their work conversely (*from top to bottom*):

**2.2.2. Measure 1.2:** *Organize for the students the activities of connection, transformation or creating mathematical representation during their thought for representation and representation for thought.*

*2.2.2.1. Object of measure*

*2.2.2.2. Scientific ground of the measure.* This measure concentrates in the second component of the mathematical representation competency.

*2.2.2.3 How to take measure*

(1). Thought process for representation; (2). Representation process for thought.

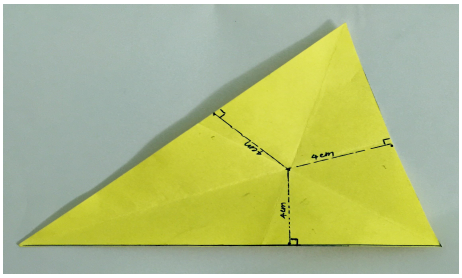
Above processes may be performed independently or alternately, insertionally.

*2.2.2.4. Attention during taking measure*
*2.2.2.5. For example*

**Example 2.9.** Teaching theorem: In a triangle, three angle bisectors meet at a point which is equally distant from three sides of such triangle. (Mathematics 7, volume 2, page.72)

Mathematical representation activity may be organized for the students with two insertional processes:

*(1). Forming theorem*: (*representation for thought*)- Perform the task: Cutting the triangle, folding the figure and define its 3 angle bisectors and look at it carefully. Draw comment and lead to theorem: *"*Three angle bisectors of a triangle will meet a point. This point is equally distant from three sides of such triangle"



*(2). Draw figure and write Supposition-Conclusion (Thought for representation).*
*(3)Finding solution by using diagram (Representation for thought):* Students use "*Diagram of finding and guess"* to connect the problem to be demonstrated with the things which have been known, provided in the task.

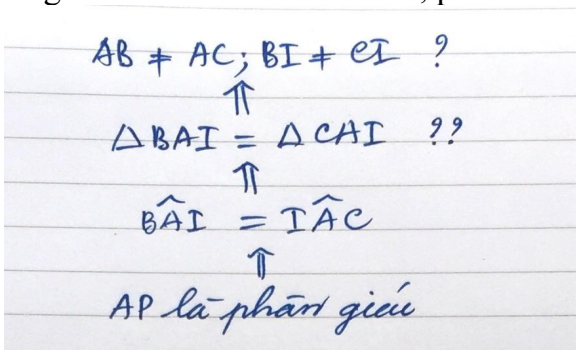



Diagram 1

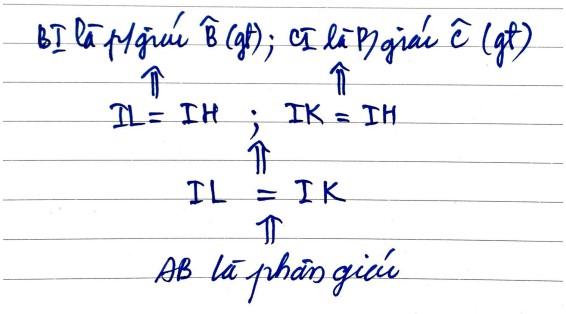



Diagram 2

In Diagram 1, students can not find solution when using definition for demonstration. Students must change and use the property of angle bisectors for demonstration (Diagram 2).

*(4) Presenting the demonstration (thought for representation).*

**2.3. Measure group 2: The measures for fostering mathematical communication competency**

**2.3.1. Measure 2.1:** *Enhancing activities including listening comprehension, reading comprehension (texts, models, diagrams, figures..) and take note (content of listening comprehension, reading comprehension) by mathematical language in teaching maths.*

2.3.1.1. *Object of the measure.*

2.3.1.2. *Scientific ground of the measure:* This measure concentrates in formation and consolidation of the first component of mathematical communication for the students.

2.3.1.3. *How to take the measure*

*a. Organization of activities for students to practise listening comprehension and taking note*

*b. Organization of activities for students to practise reading comprehension and taking note*

*c. Organization of activities for students to have skill in presentation (speak and write) using mathematical language accurately and effectively.*

2.3.1.4. *Attention during taking measure*

2.3.1.5. *For example*

***Example 2.17.*** Using 2 the same pieces of hard cover. On each piece, to draw the same line segment AB (equal) so that on only one piece there is one point M on AB; K and H are outside AB Group 1 has the piece which has line segment AB and the points M, H, K (figure 2.10a);

Group 2 has the piece which has only line segment AB and has no the points M, H, K (figure 2.10b) *Request:* Group 1 has to write a notice and send to Group 2.

According to this notice, Group 2 may define the right position of the points M, H, K on the hard cover, so that when the 2 hard covers fully coincide on each other the three points M, H, K will also fully coincide.

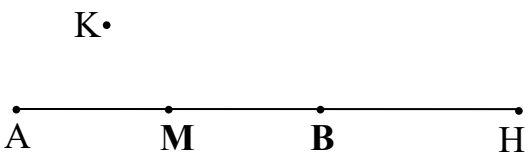


Figure 2.10a

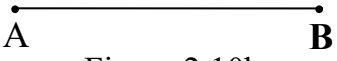


Figure 2.10b

**2.3.2. Measure 2.2**. *Guiding students to form language in mathematical speaking or writing in teaching mathematical concepts, theorems, rules or methods.*

*2.3.2.1. Object of the measure*

*2.3.1.2. Scientific ground of measure:* This measure is to form the second component of mathematical communication for the students.

*2.3.1.3. How to take measure*

*(1) Forming mathematical vocabulary and competency to master it in the close relationship with mathematical language for the students during teaching mathematical concept.*

*(2) Practising the competency to form language by observation, analysis of objects, relationship, variations in teaching theorems, rules or methods.*

*2.3.1.4. Attention during taking measure.*

*2.3.1.5. For example.*

**2.4. Measure group 3: Fostering both Mathematical representation and Mathematical communication competencies**

**2.4.1. Measure 3.1.** *Building, selection and organization for the students to take part in the mathematical representation and mathematical communication activities during solution of the mathematized problems.*

*2.4.1.1. Object of the measure.*

*2.4.1.2. Scientific ground of the measure:* This measure is formed according to paragraph 2.1.1, fostering for the students the second and third component of the mathematical representation and mathematical communication competencies.

*2.4.1.3. How to take the measure*

*a. Building and selection of mathematized problems:*

*b. Organizing mathematical representation and mathematical communication activities during solving mathematical problems:*

Activities: summarizing mathematical problem; building solution; presenting solution; exchange and share opinions; comment, evaluation.

*2.4.1.4. Attention during taking measure*

*2.3.4.5. For example:*

***Example 2.28.*** Case *"Trip of experience"*.

*Hoang Hoa Tham Junior High School organized a trip of experience for Grade 6 students in Ta Van, Sa Pa district. If total students of Grade 6 who take part in the trip are distributed in 24-seat or 36-seat bus there will be 12 students in redundancy. It is known that number of the students of Grade 6 is from 100 to 200.*

*Question a. Calculate the number of students of Grade 6 who take part in the trip?*

*Question b. If there is only 24-seat or 36-seat bus that how many buses of each kind need to be hired?*

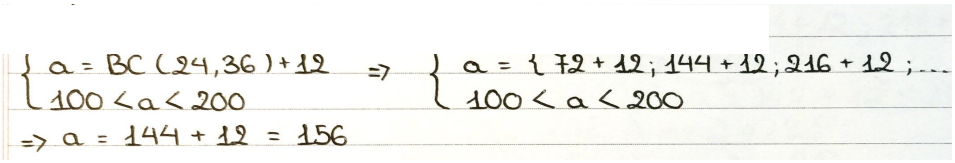
*Question c. It is known that the charge for 24-seat bus is VND 2,500,000/1 bus; the charge for 36-seat bus is VND 3,300,000 /1 bus.* Find the cheapest hire plan for this trip of

experience.

***Comment:*** Result of solving the case " Trip of experience"

a. Some groups of students found a (the number of students of Grade 6 )by trying error test with the numbers in (100, 200) which exactly divide by 24 and 36 but there are 12 students in redundancy. Other groups knew how to use mathematical symbols to define a = BC(24,36) + 12. and a belongs to (100, 200), as follows:

a. Call total students of Grade 6 is a: a ∈ **N***. We have:




b. Take total students of Grade 6 divide by 24 or 36 and use another one bus for the students in redundancy. Result: it is in need of 7 buses with 24 seats and 5 buses with 36 seats.

c. There are many ways to find the cheapest hire plan. Some groups are not flexible enough when they only compare the hire charge for 5 buses with 36 seats (VND 16,500,000) and 7 buses with 24 seats (VND 17,500,000) and made conclusion that the plan of 5 buses with 36 seats is the cheapest. However, Quynh's group made the plan based on following comparative table:

| 24-seats (bus) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 36-seats (bus) | 5 | 4 | 3 | 3 | 2 | 1 | 1 | 0 |
| Hire charge (thousand VND) | 16,500 | 15,700 | 14,900 | 17,400 | 16,600 | 15,800 | 18,300 | 17,500 |

Answer: If hiring 2 buses with 24 seats and 3 buses with 36 seats that the hire charge will be cheapest that is VND 14,900,000.

Clearly that in this case using table for representing the relationship between the number of buses and hire charge is the best plan.

**2.4.2. Measure 3.2:** *Organizing interactive learning activities (group, pair or discussion) for dealing with learning tasks with diversified answers is practical with many ways of representation suitable to the students in cognition, practice, memorization and mathematical communication.*

***2.4.2.1. Object of the measure***

***2.4.2.2. Scientific ground of Measure:*** Teaching to form and develop mathematical competency aiming to help students to be able to know mathematical relations latent in the diversified tasks. According to interaction-development teaching, it is in need to encourage students to talk about their learning tasks, using language to describe the process to get cognition; learn through the interactive activities, team-work, cooperate to solve the diversified issues of the task, close to life, share knowledge with friends, accept different thinking and viewpoints...This measure focuses on exploitation and organizing interactive activities by solving the tasks which require different mathematical representation aiming to form and develop the third component of mathematical representation and mathematical communication competencies.

***2.4.2.3. How to take measure:***

*a. Building learning task:* The open exercises, tasks have many solutions of different

level; The task provides may opportunities for the students to communicate, exchange opinions, select and change the mathematical representation.

*b. Organization for implementation:*

Step 1: Teachers give time for the students to read (listen) and summarize it using mathematical language.

Step 2: Require the students to describe the plan, solution (using mathematical language, terms, marks, diagrams, charts, drawings,....) before presentation in full.

Step 3: Presenting solution, explanation, argument, demonstration.

Step 4: Comparison, analysis of strong and weak points of the solutions.

***2.4.2.4. Attention during taking the measure***

***2.4.2.5. For example:***

***Example 2.36.*** (Mathematics for Grade 6) Students in pair to solve the task:

Draw $\widehat{xOy} = 120^0$, draw a ray Oz in the middle of the two rays Ox, Oy. Find the way to measure only once to define the measurement of the two angles xOz, zOy. How many ways to measure like that?

***Comment*:** If using the property "*If the ray Oz is between two rays Ox and Oy then* $\widehat{xOz} + \widehat{zOy} = \widehat{xOy}$ " we have two solutions 1, 2. And another property: "*Two supplementary angles add up to* $180^0$*"* we have solutions 3, 4 and 5. This task provides the students opportunities to practise communication during discussion, exchange and share the different solutions, particularly:

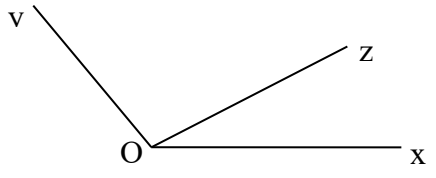



Figure 2.24a

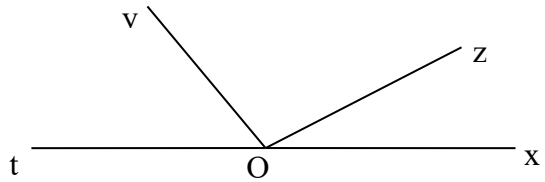



Figure 2.24b

See Figure 2.24a, it is given that:

Ray Oz is between Ox and Oy, so: $\widehat{xOz} + \widehat{zOy} = \widehat{xOy} = 120^0$

*Method 1:* Measure angle xOz, calculate difference: $120^0$ - $\widehat{xOz}$ and we have the measurement of yOz

*Method 2:* Measure angle yOz, calculate difference: $120^0$ - $\widehat{yOz}$ and we have the measurement of xOz

See Figure 2.24b: Two supplementary angles add up to $180^0$, we can draw another ray Ot which is the opposite ray of Ox (or ray Oy or ray Oz). Then:

*Method 3:* Measure angle zOt, calculate difference $180^0$ - $\widehat{zOt}$ , we have the measurement of $\widehat{xOz}$. Continue to calculate: $120^0$ - $\widehat{xOz}$ we have the measurement of yOz.

*Method 4 and 5*: similarly to *Way 3* by drawing another ray Ou; Ov which are opposite to rays Oy, Oz respectively.

Additionally, the teacher may give students another task as follows: *Draw acute angle xOy. Draw ray Oz between two rays Ox, Oy. How to know measurement of three angles xOy, xOz, yOz by measuring only two times.*

Organize for the students to make comment, evaluation about the implementation. It is in need to help students practise the description, presentation of the solution using diagram, drawing (especially in solution of geometrical tasks).

**2.4.3. Measure 3.3.** *Building and organizing learning activities under the project by strengthening mathematical representation and mathematical communication in each step of the project.*

***2.4.3.1. Object of the measure***

***2.4.3.2. Scientific ground of measure:*** This measure is built based on paragraph 2.1.3, focusing on the learning activities with a view to fostering the students with the second and third component of mathematical representation and mathematical communication.

***2.4.3.3. How to perform the measure.*** 3-step process under the Project.

*Step 1: Make plan; Step 2: Carry out the Project; Step 3: Assemble results.*

***2.4.3.4. Attention during taking measure***

***2.4.3.5. For example.***

***Example 2.39. Project: Park*** (Group including 10-12 students)

*Nhac Son Park (Hoang Lien road, Lao Cao city) is a small park but its has unique and harmonious architectural characteristics with the lake as silk strip meandering around (picture). An event will be held here with a rectangle stage floating on the lake so that its width calculated in the direction of the width of the lake and the distance from the lake bank to the outer side of the stage is at least 3m. Define the suitable location and the width of such stage.*

***Comment:*** HS should have certain knowledge of the actual situation and location to choose a position "suitable" for the stage design. Then, use the knowledge of equality of the right triangles and skill in using the measures such as goniometers, guide posts, tape measurer... to practise measurement of the distance between two points A and B (in two lake banks) .

Process and the way to organize learning under the Project with above 3 steps.

In this Project, mathematical representation activities are focused on following aspects: Field survey, sketch drawing: Describe the position of the lake bank, rectangle stage .... based on which to define the position of the points to be measured: width of the lake (the narrowest position), measure, calculate to define the width of the stage. Make report, modelize the actual factors using geometry. Describe the measurement and calculate based on such model and demonstrate the indirect measurement that define the distance which needs to be measured (based on the model). Based on the measured figures, conditions of the task and geometric model, calculate and define the width of the stage.

Mathematical communication activity focuses on following contents: Read and understand the requirements of the task, discuss and make the solution plan, persuadable presentation, reasonable explanation about the area to be selected ("suitability"), collected presentation, scientific and reasonable argument of the measurement and calculation process to have the final result.

**Conclusion of Chapter 2**

Based on the theoretical and practical researches stated in Chapter 1, we focused on the research and recommend the measures for fostering mathematical representation and mathematical communication competencies for the students in teaching mathematics for grade 6 and 7.

Based on the typical manifestations of each component of the mathematical representation and mathematical communication competencies, we make the measures for fostering and developing this competency for the students. The result is that Chapter 2 recommended 7 measures of 3 groups of measures: (1) *Group of measures for fostering mathematical representation competency, including 2 measures*; (2) *Group of measures for fostering mathematical communication competency, including 2 measures*; *(3) Group of measures for fostering both mathematical representation and mathematical communication competencies, including 3 measures*. Each measure is defined by its *name*, *object*, *scientific ground; way to perform; Attention during taking measure and the examples as illustration* in the mathematical program for Grade 6, 7.

For each measure, we provide guidance for the teachers to organize activities for the students during teaching mathematics of Grade 6 and 7, with a view to focus on the certain manifestations of each component of the mathematical representation or mathematical communication competencies; point out the opportunities and conditions for taking the measure in the context where time is not much, and there are strict requirements in respect of the object, content, curriculum. The recommended measure has been always taken into account to ensure its scientific nature, practicability, suitability to the students of Grade 6 and 7.

# Chapter 3
# PEDAGOGIC EXPERIMENTATION

## 3.1. Purpose and requirements

**3.1.1. Purpose:** Pedagogical experimentation is conducted to test the scientific hypothesis. It initially confirms the feasibility and effectiveness of pedagogical measures proposed in Chapter 2 through teaching practice.

### 3.1.2. Experimental requirements

## 3.2. Experimental tasks

## 3.3. Principles of experimental organization

## 3.4. Time, subject of experiment

### *3.4.1. Pedagogic experimentation 1*

### *3.4.2. Pedagogic experimentation 2*

## 3.5. Process of experimentation organization

### *3.5.1. Process of experimentation*

## 3.6. Content of experimentation

### *3.6.1. Content of experimental teachings*

#### *3.6.1.1. Experimentation 1*

*a. Experimental period:* Total experimental periods: 10 periods.

*b. About the experimented pedagogical methods:* Including Measures 1 and 2

*c. Learned Lessons:*

(1) Teacher should give initial instructions for students on how to ask and answer, brief, and record the mathematical contents.

(2) Care about homework preparation of students, including consider mathematical communication and representation as a compulsory task.

(3) Focus on all students when organizing teaching.

(4) Spoken language of teacher should be brief, coherent, easy to understand, in coordination with modeling tasks (drawing, indications on the diagram, chart, ...)

(5) Need to study, recommend measures improving both competencies of mathematical representation and communication by mathematical language for students in teaching mathematics.

***3.6.1.2. Experimentation 2***

*a. About experimental period:* 36 periods (in which 12 periods of specialized topics)

*a. About experimented pedagogical measures:* Including 3 measure groups 1, 2 and 3 with 7 specific measures.

**3.6.2. Content of tests before and after experimentation**

***3.6.2.1. Test before experimentation***

***3.6.2.2. Test after experimentation***

**3.7. Result of experimental process**

**3.7.1. Quantitative assessment**

***3.7.1.1. Measures:*** *Pedagogical observance*

***3.7.1.2. Result:*** Alter a time of experimental teaching, students can understand the core content to record, read, summarize the main ideas, pose the questions and answers to find demonstration direction. Students can read illustration, graphs, charts, etc., recognize mathematical relationships expressed in the forms of mathematical language.

Students need to express their political views and use tree diagrams, thinking charts to summarize a simple mathematical content to perform with the increasing level of complexity. Students of experimental classes become more confident in learning, boldly offer ideas, have familiarity and ability to use diagrams and models to summarize or describe ideas and solutions in an oral presentation or a written presentation.

Besides, teachers need to improve skills and be active in organizing diversified language activities in each teaching period. Teachers express the idea of given measures and know how to exploit, make use of situation suitable for mathematical communication and mathematical representation. Be sensitive with language in teaching, especially mathematical language used by students during learning process and timely correct by reasonable and effective impacts.

**3.7.2. Qualitative assessment**

***3.7.2.1. Measures:*** Data of exam marks converted to the 5-level scale of mathematical representation and 5-level of mathematical communication settled by method of mathematical statistics.

***3.7.2.2. Experimental result***

*a) Result of test before experimentation 1*

*b) Result of test after experimentation 1*

*c) Result of test before experimentation 2*

*d) Result of test after experimentation 2*

*Table 3.4a: Distribution of competency assessment mark on mathematical representation of experiment group and control group after experiment round 2*

| Group | Mark $x_i$ / No. of students | **1** | **2** | **3** | **4** | **5** | $\overline{X}$ | $\mathbf{S^2}$ |
|---|---|---|---|---|---|---|---|---|
| Control | 76 | 8 | 11 | 40 | 11 | 6 | 2.95 | 1.018 |
| | (%) | 10.5 | 14.5 | 52.6 | 14.5 | 7.9 | | |
| Experiment | 81 | 2 | 4 | 36 | 25 | 14 | 3.56 | 0.922 |
| | (%) | 2.5 | 4.9 | 44.4 | 30.9 | 17.3 | | |

*Table 3.4b: Distribution of competency assessment mark on mathematical communication of experiment group and control group after experiment round 2*

| Group | Mark $x_i$ / No. of students | 1 | 2 | 3 | 4 | 5 | $\overline{X}$ | $S^2$ |
|---|---|---|---|---|---|---|---|---|
| Control | 76 | 5 | 14 | 39 | 13 | 5 | 2.99 | 0.945 |
| | (%) | 6.6 | 18.4 | 51.3 | 17.1 | 6.6 | | |
| Experiment | 81 | 1 | 5 | 34 | 23 | 18 | 3.64 | 0.928 |
| | (%) | 1.2 | 6.2 | 42.0 | 28.4 | 22.2 | | |

***Comparing variance of two groups,*** it can be found that: The difference between variance of experiment group and control group is not significant for statistics.

***Comparing average mark of two groups,*** it can be concluded that: The average competency assessment mark of mathematical representation and communication after experiment 2 of experiment group is higher than that of control group.

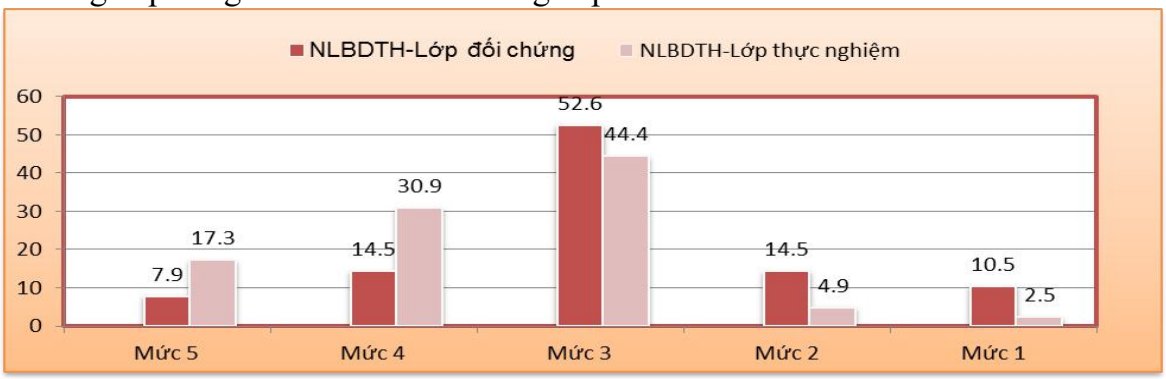



Chart 3.6a. *Chart of mathematical representation competency level of students after experiment 2 of experiment group and control group*

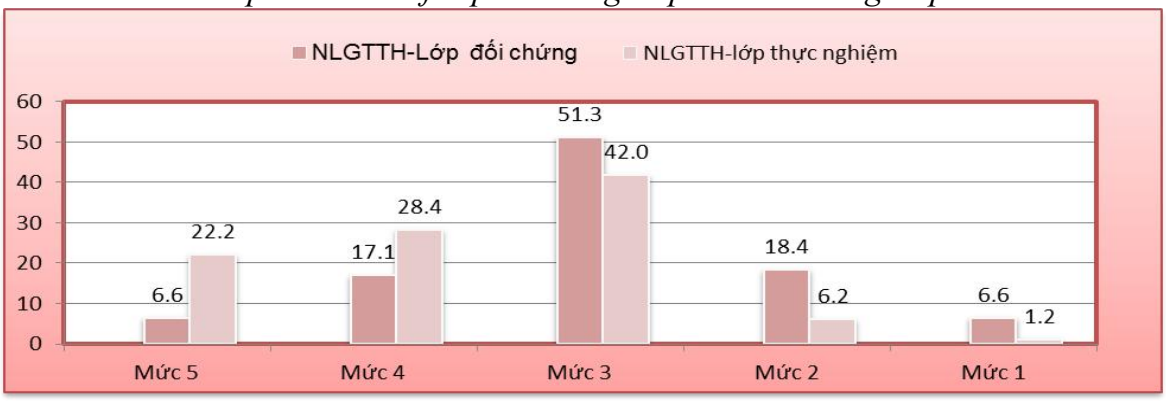



Chart 3.6a. *Chart of mathematical communication competency level of students after experiment of experiment group and control group*

## Conclusion of Chapter 3

Chapter 3 presents the experiment process and assessment on experiment result of three pedagogical measure groups on fostering mathematical representation and mathematical communication competencies for students of grade 6, grade 7 (with 7 specific measures). The result of two independent experiments has made clear on theoretical and practical issues on mathematical representation and communication competencies of

students. In addition, it confirms the efficiency and feasibility of the proposed pedagogical measures.

The thesis selects experiment group and control group of the same learning ability and competency level of mathematical representation and mathematical communication. For experiment group, in teaching with measures of fostering mathematical representation and mathematical communication competencies proposed in Chapter 2. For control group, during teaching process, there are also mathematical representation and mathematical communication spontaneously. However, due to the habit of focusing on academic knowledge and problem-solving results, those processes have not been effectively used to foster mathematical representation and mathematical communication competencies for students. Therefore, students of control class face with more difficulties and get puzzled when settling language before solving mathematical problems, capacity of describing solution before representation, capacity of changing language to facilitate thinking and communication, etc. Students can hardly solve mathematical problems in unfamiliar situations, not similar to known theory, etc. For experiment group, students can perform better with situations of this type and get higher result than control group.

Testing and observation, analysis, consideration on student's notebook show that: students of experiment group can present mathematical content in a scientific, brief and logic manner. Students of control group present mathematical content in a wordy, illogic and noncreative manner. Students of experiment group are more flexible in using mathematical charts and have a deeper and fuller knowledge on mathematical concepts and relations.

It can be affirmed that, performing the measures proposed in the thesis has formed and developed mathematical representation and mathematical communication competencies for students, thereby fostering the love for mathematics, strengthening proactive-ness, self-awareness, activeness in learning, improving performance result of mathematics for students of lower grades of junior high school. In summary, the purpose of experiment has been completed, the feasibility and effectiveness of measures begin to be affirmed, scientific hypothesis of the thesis is acceptable on the aspect of practice.

## CONCLUSION

The thesis has fulfilled the set-out research content and task, established pedagogical measures to foster mathematical representation and mathematical communication competencies for students through teaching Mathematics of grades 6 and 7, with the aim to improve performance result of Mathematics for students. The thesis obtains the main results as follows:

1. Giving an overview of mathematical language, mathematical representation, and mathematical communication. To give general concept of using mathematical language and mathematical language using competency, on that basis, establishing concept of mathematical representation and mathematical communication competencies, determining components, specific expressions and levels of mathematical representation and mathematical communication competencies of junior high school students.

2. Analyzing mathematical language in curriculum, textbook of mathematics grade 6, grade 7. Studying the actual status of teaching, fostering mathematical representation and mathematical communication competencies for junior high school students in current time, analyzing causes as the basis to recommend measures of fostering mathematical representation and mathematical communication competencies for students in teaching Mathematics grade 6, grade 7.

3. Determining 3 principles of orientation for establishing measures to foster mathematical representation and mathematical communication competencies. On that basis, to establish three measure groups, including 7 specific measures to foster mathematical representation and mathematical communication competencies. With each measure, clearly describe purpose, scientific foundation, content, guidance on performance, notes during performance, and illustration examples for each measure. Details are as follows:

***Measure group 1: Fostering mathematical representation competency***

*Measure 1.1 Organize activities of identifying, understanding and using right representation forms on subjects, relations and steps of mathematical changes.*

*Measure 1.2: Organize activities of connecting, changing or creating mathematical representation during thinking for representation and representation for thinking.*

***Measure group 2: Fostering mathematical communication competency***

*Measure 2.1: Enhancing activities of listening, reading comprehension (documents, models, diagrams, pictures, etc.) and note-taking (content of listening and reading comprehension) by mathematical language in teaching mathematics.*

*Measure 2.2: Instructing students the process of creating mathematical oral or written language products in teaching concepts, theorems, rules, and mathematical methods.*

***Measure group 3: Fostering both mathematical representation and communication competencies***

*Measure 3.1. Establishing, selecting and organizing for students to perform mathematical representation and mathematical communication competency during settling mathematical situations*.

*Measure 3.2. Organizing activities of interactive work (in group, in pair or common discussion) in performing learning tasks diversified in answer, with practical element, having many representations suitable with students in mathematical awareness, memorization, and communication.*

*Measure 3.3. Organizing learning under project by enhancing mathematical representation and mathematical communication activities in each step of project performance.*

4. Organizing experiment to illustrate feasibility and effectiveness of the proposed pedagogical measures.

Basing on results of research, it can be affirmed that the target of research has been reached, the duties of research has completed and scientific assumptions are acceptable. The thesis has affirmed that measures of fostering mathematical representation and mathematical communication competencies are efficient and feasible, increasing results of learning mathematics, developing capacity of logic thinking and using accurate language for students of secondary schools. On the other hand, pedagogic measures have shown clearly the teaching opinion in activities and by self-aware, active, and creative activities of students; thereby, establishing and developing the students' mathematical representation and mathematical communication competencies in particular and mathematical competency in general. Concurrently, contributing to clarify the oriented renovation of teaching in the direction of developing competency for learners, establishing the active, self-respecting and confident characteristics of students during the process of occupying knowledge, developing the capacity of efficient self-learning towards lifelong study of students.